Lokale initiatieven Doctoral Schools (DS/ST)

Summer school on "Hyperbolic conservation laws"

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Many processes are described by physical laws expressing the conservation of mass, momentum, or energy. Commonly, the related mathematical models are systems of partial and ordinary differential equations. One illustrative example is the air flow around planes under standard flight conditions. In mathematical terms, this flow is modelled by the Navier-Stokes or the Euler equations, which form a particular set of mathematical conservation laws.

The focus of the summer school is on *hyperbolic conservation laws*. The mathematical treatment of conservation laws is a complex and challenging task. In comparison to other types of equations, the hyperbolic conservation laws may have multiple solutions, which may become non-smooth. This makes both the mathematical analysis and the numerical approximation quite cumbersome. Most of the available results are restricted to simplified and hence non-realistic cases, like one-dimensional problems. In the general situations the existence of solutions is still an open research question. Also, in case of multiple solutions, entropy criteria have to be defined to select the physically relevant solution.

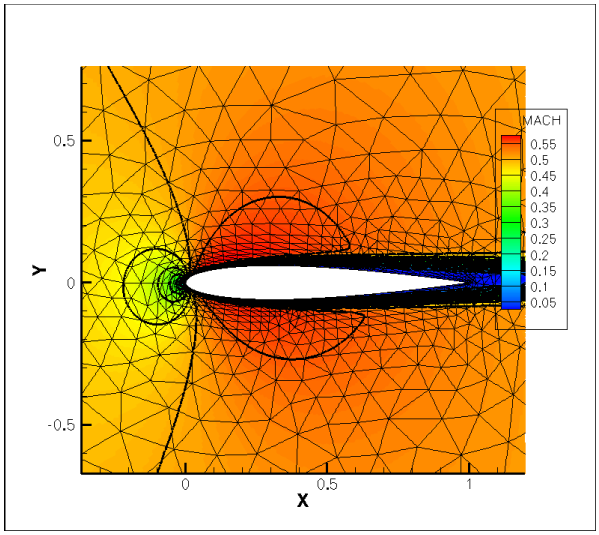

Fig. 1: The Mach-number (a normalized velocity) distribution around an idealized airfoil computed as the solution to the Navier-Stokes equations.

The goal of this summer school is to give the participants a basic introduction to the analysis and the numerics of hyperbolic conservation laws. The focus will be more on an introduction to the concepts including their applications rather than a complete and rigorous discussion of the theory. The summer school will therefore be amenable to mathematicians, engineers and scientists and will be presented in a genuine interdisciplinary context. The summer school will be open to participants having a basic knowledge in calculus and linear algebra courses as they are taught to engineers and scientists. All sessions will be hands-on, meaning that sufficient time for exercises and their discussion is planned.

The duration of the summer school is estimated to be three days, its tentative structure is as follows:

Day 1. First an introduction to the analysis of conservation laws is given. In particular, the theory of scalar, non-linear hyperbolic conservation laws is discussed. As a building stone for both the analysis and the numerics, the so-called Riemann problem is discussed in detail.

Day 2. The content of day two is twofold. In the morning, the role of conservation laws in computational fluid dynamics is presented. First the equations describing fluid flow are derived, based on fundamental physical principles. In the afternoon, an introduction to numerical methods for the treatment of conservation laws is given. Based on the Riemann-problem, the participants will implement a conservation law solver in Matlab.

Day 3. Day three builds upon the content of days one and two and introduces the “Discontinuous Galerkin method”, a modern state-of-the-art solution technique for nonlinear systems of conservation laws. The open-source tool Netgen/NGSolve with its easy-to-handle Python interface is introduced to the participants and used in the exercises, so that discussed theoretical effects can be observed on the spot.

Day three ends with a dinner, giving the participants the possiblity to discuss matters and to build a network.